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Question Paper Code : 30877

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2024.

Fourth Semester

Mechanical Engineering

MA 8452 – STATISTICS AND NUMERICAL METHODS

(Common to : Automobile Engineering/Mechatronics Engineering/
Production Engineering/Robotics and Automation)

(Regulations 2017)

Time : Three hours

Maximum : 100 marks

(Use of statistical table is permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Derive the mean of the sampling distribution of the sample mean.
2. Give examples for Type-I and Type-II errors.
3. Write down the model equation for one-way classification.
4. Write down the model equation for randomized block design.
5. Write the Newton-Raphson formula for finding the value of $\sqrt{N}$.
6. Write the process of finding the eigenvalues by Jacobi method for symmetric matrices.
7. Write down the linear Lagrange interpolating polynomial formula that passes through the points (x_0, y_0) and (x_1, y_1) .
8. Evaluate $\int_0^1 x^2 dx$ using trapezoidal rule.
9. Given that $y' = 1 - y$ and $y(0) = 0$. Find $y(0.1)$ by Euler method.
10. Write down the Milne's predictor corrector formula.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A random sample of size $n=100$ is taken from a population with $\sigma=5.1$. Given that the sample mean is $\bar{x}=21.6$, construct a 95% confidence interval for the population mean μ . (8)
- (ii) Let X be a normal random variable with variance $\sigma^2=0.0001$. Using a sample of size $n=35$ with mean $\bar{x}=0.343$, test the hypothesis $\mu=0.34$ against the hypothesis $\mu \neq 0.34$ at the 0.05 level of significance. (8)

Or

- (b) (i) From a population, a sample is given as 2.4, 2.9, 2.7, 2.6, 2.9, 2.0, 2.8, 2.2, 2.4, 2.4, 2.0, 2.5. Test the null hypothesis $\mu=2.25$ against the alternative hypothesis $\mu > 2.25$ at the 0.025 level of significance. (8)
- (ii) The following table gives the number of accidents that occurred during the various days of a week.

Day	:	1	2	3	4	5	6
Accidents	:	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week. (8)

12. (a) Let A, B, C denote 3 makes of computers. Samples of size 5 are selected from each make and the frequency of repair during the first year of purchase is observed as given below.

A	5	6	8	9	7
B	8	10	11	12	4
C	7	3	5	4	1

Test the hypothesis that the 3 makes of computers do not differ in the durability. (16)

Or

- (b) Three varieties of a crop are tested in a randomized block design with four replications, the layout being as given below such that the rows represent the blocks and the columns represent the varieties of the crop. The yields are given in kilograms.

Blocks	A	B	C
1	47	49	48
2	51	49	53
3	49	52	52
4	49	50	51

By shifting the origin to 50, analyse for significance. (16)

13. (a) (i) Apply Gauss elimination method to solve the equation
 $x + 4y - z = -5$; $x + y - 6z = -12$; $3x - y - z = 4$. (8)

(ii) Find by Newton's method the real root of the equation
 $3x = \cos x + 1$, correct to four decimal places. (8)

Or

(b) (i) Apply the Gauss-Seidel iteration method to solve the equations
 $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$. (8)

(ii) Determine the largest eigenvalue and the corresponding eigen-
vector of the matrix $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ by using Power method. Let $X_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. (8)

14. (a) (i) From the following table, find $f(x)$ by Newton's divided difference
formula. (8)

$x:$ 1 2 7 8

$f(x):$ 1 5 5 4

(ii) Find the value of $\cos(1.74)$ using numerical differentiation from the
following table: (8)

$x:$ 1.7 1.74 1.78 1.82 1.86

$\sin x:$ 0.9916 0.9857 0.9781 0.9691 0.9584

Or

(b) (i) Use the Trapezoidal rule, estimate the integral $\int_0^2 e^{x^2} dx$ taking 10
subintervals. (8)

(ii) Evaluate the integral $\int_0^1 \frac{x^2}{1+x^3} dx$ using Simpson's 1/3 rule. Compare
the error with the exact value. (8)

15. (a) (i) Using Taylor series method, find correct to four decimal places, the
value of $y(0.2)$, given $\frac{dy}{dx} = 1 - 2xy$ and $y(0) = 0$. (8)

(ii) Given $y' = x^2 - y$, $y(0) = 1$, find correct to four decimal places the
value of $y(0.1)$ by using Improved Euler method. (8)

Or

- (b) (i) Apply the Runge-Kutta fourth order method to find an approximate value of y when $x=0.2$ given that $\frac{dy}{dx} = x + y$ and $y=1$ when $x=0$. (8)
- (ii) Given that $5xy' + y^2 - 2 = 0$ with $y(4)=1$, $y(4.1)=1.0049$, $y(4.2)=1.0097$, $y(4.3)=1.0143$, $y(4.4)=1.0187$. Using Milne's method, find $y(4.5)$. (8)